Engineering Notes

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Minimum-Ballistic-Factor Flat-Nosed Missiles

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Introduction

IN a recent paper, Jain and Tawakley[1] discussed the problem of finding the geometries of pointed missiles of minimum ballistic factor for the case when the shapes are continuous and have positive slopes everywhere. In another paper Tawakley and Jain[2] extended the results to the case where the nose radius of the missile has a specified value, not necessarily zero, and showed that where the surface area S and diameter d are known in advance, an analytical solution can easily be obtained. It was proved that the result was valid up to a certain value of the friction factor $k=4c_f\ (S^3/\pi^3 d^6)$, (c_f being the surface-averaged skin-friction coefficient, assumed to be constant), referred to as the critical value. In the present paper the result has been extended to the case where the value of k exceeds the critical value and the optimal shapes consist of analytic curves having finite positive slopes followed by cylinders.

Necessary Conditions for Extremal

Under the assumptions that the flow is along the axis of the missile, the pressure distribution is Newtonian, and the surface-averaged skin-friction coefficient is constant, it was shown in Ref. 2 that the following functional expression has to be minimized in order to find the minimum-ballistic-factor missile shape

$$\frac{S'^3}{\pi^3 d^5}\frac{D}{qV}=\frac{I_1 I_2^3}{I_3}+K'\frac{I_2}{I_3} \tag{1}$$

where

$$I_1=\int_0^1 YY'^3\mathrm{d}X \qquad I_2=\int_0^1 Y\mathrm{d}X \qquad I_3=\int_0^1 Y^2\mathrm{d}X \tag{2}$$

and

$$S'=S-\pi(d/2)Y_0^2$$

$$K'=4c_f\frac{S'^3}{\pi^3 d^6}+2\frac{S'^2}{\pi^2 d^4}Y_0^2 \tag{3}$$

In the above, X and Y are the dimensionless abscissa and ordinate of the missile ($X=x/l$, $Y=(2/d)y$, l being the length and d the diameter) and the subscript zero refers to the nose radius.

Since the possibility of the missile having a zero-slope shape is being considered, $Y'\geq 0$. This inequality may be expressed as

$$Y'-Z^2=0 \tag{4}$$

where $Z(X)$ denotes a real variable.

According to theory,[1] the necessary condition for extremizing the functional expression in Eq. (1) is identical to that for extremizing a new functional of the form

$$J=\int_0^1 F(X,Y,Y',Z,\mu_j,\nu)\mathrm{d}X \qquad (j=1,2,3)$$

where F denotes the fundamental function

$$F=\mu_1 YY'^3+\mu_2 Y+\mu_3 Y^2-\nu(Y'-Z^2) \tag{5}$$

Here $\nu=\nu(X)$ is a variable Lagrange multiplier and μ_1, μ_2, μ_3 are certain constant multipliers determined in Ref. 2 as

$$\mu_1=\frac{\lambda_1}{1+K'\lambda_1\lambda_2^2} \qquad \mu_2=\frac{3\lambda_2+K'\lambda_1\lambda_2^3}{1+K'\lambda_1\lambda_2^2} \qquad \mu_3=-\lambda_3 \tag{6}$$

where

$$\lambda_j=1/I_j \qquad (j=1,2,3) \tag{7}$$

From the calculus of variations it is known that the extremal solution must satisfy the Euler equations

$$6\mu_1 YY'Y''+2\mu_1 Y'^3-\mu_2-2\mu_3 Y-\nu'=0 \tag{8}$$

$$\nu Z=0$$

The second Euler equation shows that the extremal arc consists of either a purely regular shape ($\nu=0$) or a combination of regular shape ($\nu=0$) and zero-slope shape ($Z=0$). In Ref. 2 only regular shapes were obtained. Since the fundamental function F does not contain the dependent variable

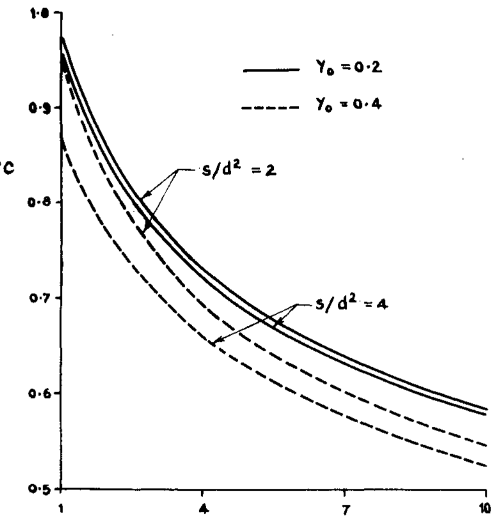

Fig. 1 Relation between the transition point X_c and the friction parameter k.

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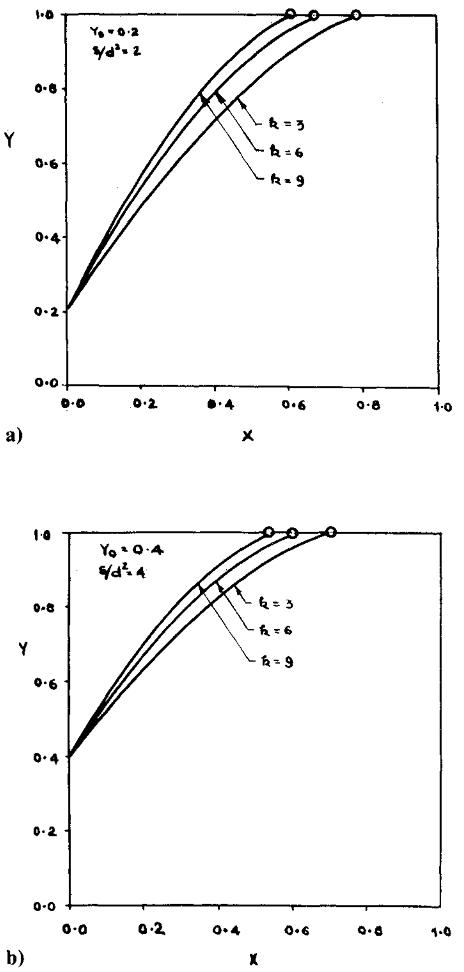

Fig. 2 Optimum shapes for given nose radius, dimensionless surface area and diameter: a) $Y_0 = 0.2$, b) $Y_0 = 0.4$.

X explicitly, the Euler equation admits the first integral

$$2\mu_1 YY'^3 - \mu_2 Y - \mu_3 Y^2 = C \tag{9}$$

where C is an integration constant.

By considering the Corner and Weierstrass conditions, it was shown by Tawakley and Jain[3] that if the surface area and diameter are known a priori, the minimal arc consists of either a regular shape only or a regular shape followed by a zero-slope shape $Y=1$. Purely regular shapes are obtained up to the value of $k = 4c_f\ (S^3/\pi^3 d^6)$ given by Ref. 2:

$$k = \frac{27}{800}(1-Y_0)^2(3+2Y_0)^2(3+2Y_0+3Y_0^2)\left(1-\frac{\pi^2 d^2 Y_0^2}{4S}\right)^{-3}$$
$$-\frac{2S^2 Y_0^2}{\pi^2 d^4}\left(1-\frac{\pi d^2 Y_0^2}{4S}\right)^{-1}$$

Now in what follows, shapes are obtained for values of k which are greater than this critical value. It was proved in Ref. 1 that when length is a free variable, $C=0$. Also, it was shown in Ref. 3 that $\mu_2 + \mu_3 = 0$. Thus from Eq. (9) we see that the minimizing curve is given by

$$\frac{X}{X_c} = 1 - \left(\frac{1-Y}{1-Y_0}\right)^{2/3} \qquad (0 \le X \le X_c)$$

$$Y = 1 \qquad (X_c \le X \le 1) \tag{10}$$

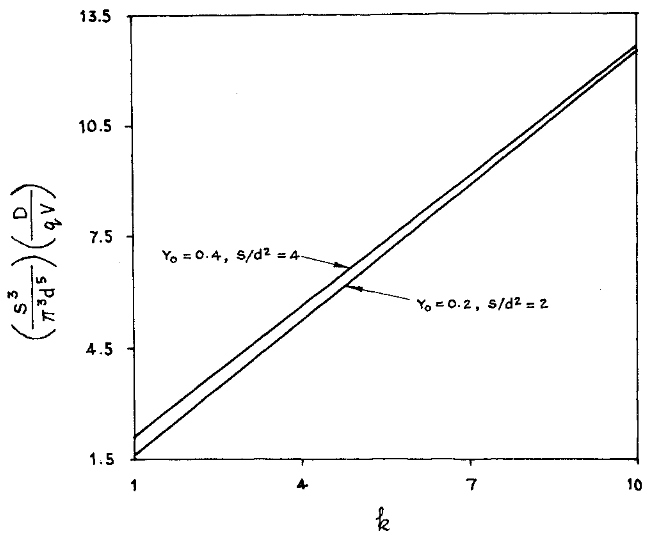

Fig. 3 Minimum ballistic factor vs the friction parameter.

where the subscript c represents the transition or corner point. Again from Eq. (9),

$$X_c = \frac{3}{2}\left(\frac{2\mu_1}{\lambda_3}\right)^{1/3}(1-Y_0)^{2/3} \tag{11}$$

Also by making use of Eqs. (2), (7), and (10) it can be deduced that

$$\frac{1}{\lambda_1} = \frac{27(1-Y_0)^3}{160X_c^2}(5Y_0+3) \tag{12}$$

$$\frac{1}{\lambda_2} = 1 - \frac{2}{5}X_c(1-Y_0) \tag{13}$$

$$\frac{1}{\lambda_3} = 1 - X_c\left(\frac{11}{20} - \frac{3}{10}Y_0 - \frac{1}{4}Y_0^2\right) \tag{14}$$

With the help of Eqs. (6, 11, 12, 13, and 14) the following equation for solving for the corner point is obtained:

$$X_c^3\left[\left(\frac{324}{5}Y_0 + 108\right)(1-Y_0)^5 + 160k\right] - X_c^2\left(324Y_0 + \frac{3564}{5}\right)(1-Y_0)^4 + X_c(405Y_0 + 1539)(1-Y_0)^3 - 1080(1-Y_0)^2 = 0 \tag{15}$$

Figure 1 represents the relation between X_c and k for known values of Y_0 and S/d^2. From a knowledge of the corner point the shapes of the minimal curves can be obtained from Eq. (10) and are illustrated in Fig. 2. Also, since λ_1, λ_2, and λ_3 are known from Eqs. (12, 13, and 14), respectively, the value of $(S'^3/\pi^3 d^5)(D/qV)$ can be calculated from Eq. (1). The result is plotted in Fig. 3.

References

[1] Jain, S.C. and Tawakley, V.B., "Optimum Missile Geometries of Minimum Ballistic Factor," *Astronautica Acta*, Vol. 18 (Supplement), 1974, pp. 119-126.

[2] Tawakley, V.B. and Jain, S.C., "Shapes of Blunt-Nosed Missiles of Minimum Ballistic Factor," *Journal of Spacecraft and Rockets*, Vol. 13, Oct. 1976, pp. 631-634.

[3] Tawakley, V.B. and Jain, S.C., "Minimum Ballistic-Factor Problem of Slender Axisymmetric Missiles," *Defence Science Journal*, Vol. 29, No. 1, 1979.